Further Maths Revision Paper 2

This paper consists of 5 questions covering CP1, CP2, FP1 and FM1.
(AS Further Maths: Q4 and 5)

1

Use L'Hospital's Rule to calculate the

$$\lim_{x\to 0} \frac{1-\cos x}{x^2}$$

$f(x) = 1 - \cos x$ $\quad$ $g(x) = x^2$

$f(0) = 0$ $\quad$ $g(0) = x^2$

By L'Hôpital's rule

$$\lim_{x\to 0} \frac{f(x)}{g(x)} = \lim_{x\to 0} \frac{f'(x)}{g'(x)}$$

$f'(x) = \sin x$ $\quad$ $g'(x) = 2x$

$f'(0) = 0$ $\quad$ $g'(0) = 0$

By L'Hôpital's Rule

$$\lim_{x\to 0} \frac{f'(x)}{g'(x)} = \lim_{x\to 0} \frac{f''(x)}{g''(x)}$$

$f''(x) = \cos x$ $\quad$ $g''(x) = 2$

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \lim_{x\to 0} \frac{\cos x}{2}$$

$$= \underline{\underline{\frac{1}{2}}}$$

2

Draw the polar curve

$$r^2 \sin 2\theta = 2c^2$$

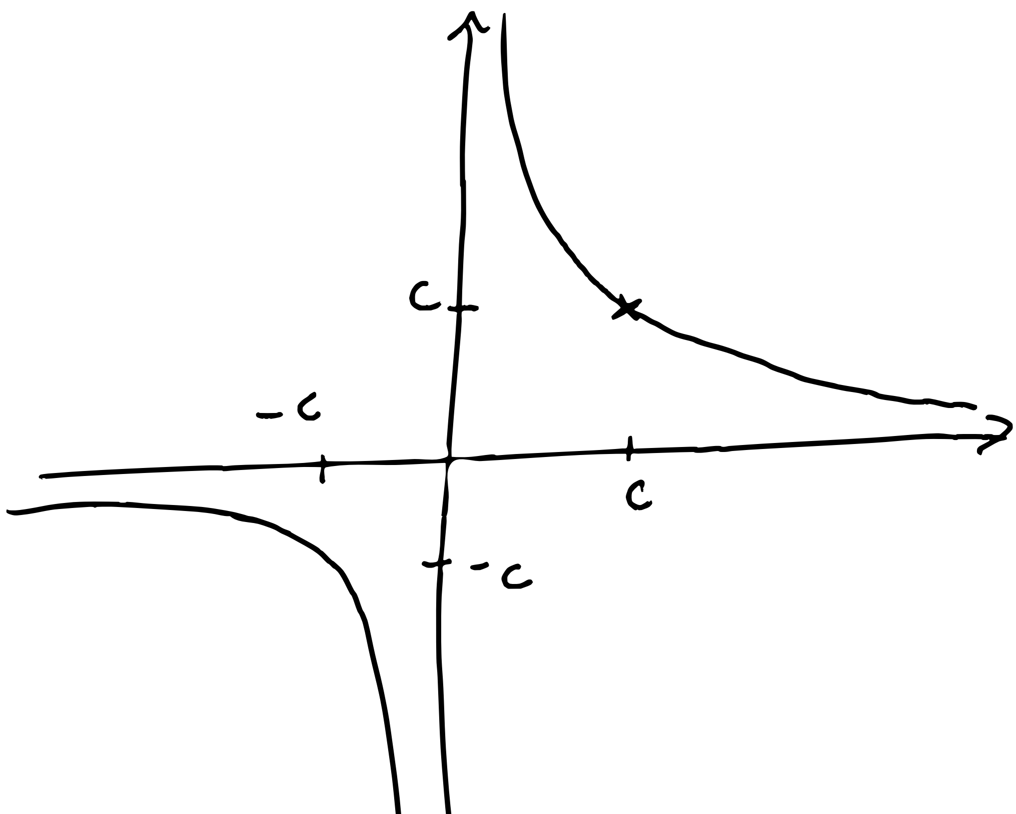
marking key points on your sketch.

$$2r^2 \sin\theta \cos\theta = 2c^2$$

$$r^2 \sin\theta \cos\theta = c^2$$

$$r\sin\theta \, r\cos\theta = c^2$$

$$\underline{\underline{xy = c^2}}$$



3

(a) Prove that

$$\frac{(2n+1)(2n+3)}{(n+1)(n+2)} - \frac{(2n-1)(2n+1)}{(n(n+1)} = \frac{2(2n+1)}{n(n+1)(n+2)}$$

(b) Hence or otherwise show that the sum of the first n terms of the series

$$\frac{3}{1\times 2\times 3} + \frac{5}{2\times 3\times 4} + \cdots$$

is

$$\frac{n(5n+7)}{4(n+1)(n+2)}$$

a)

$$\frac{n(2n+1)(2n+3) - (2n-1)(2n+1)(n+2)}{n(n+1)(n+2)}$$

$$= \frac{(2n+1)(2n^2+3n - (2n^2+3n-2))}{n(n+1)(n+2)}$$

$$= \frac{(2n+1)(2)}{n(n+1)(n+2)}$$

b)

$r=1$ $\quad \frac{3}{1\times2\times3} = \frac{(3)(5)}{2(2)(3)} - \frac{(1)(3)}{2(1)(2)}$

$r=2$ $\quad \frac{5}{2\times3\times4} = \frac{(5)(7)}{2(3)(4)} - \frac{(3)(5)}{2(2)(3)}$

$r=3$ $\quad \frac{7}{3\times4\times5} = \frac{(7)(9)}{2(4)(5)} - \frac{(5)(7)}{2(3)(4)}$

$\vdots$

$r=n-1$ $\quad \frac{2n-1}{(n-1)n(n+1)} \quad \frac{(2n-1)(2n+1)}{2n(n+1)} - \frac{(2n-3)(2n-1)}{2(n-1)(n)}$

$r=n$ $\quad \frac{2n+1}{n(n+1)(n+2)} \quad \frac{(2n+1)(2n+3)}{2(n+1)(n+2)} - \frac{(2n-1)(2n+1)}{2n(n+1)}$

$$\sum_{1} \frac{2n+1}{n(n+1)(n+2)} = \frac{(2n+1)(2n+3)}{2(n+1)(n+2)} - \frac{3}{4}$$

$$= \frac{2(2n+1)(2n+3) - 3(n+1)(n+2)}{4(n+1)(n+2)}$$

$$= \frac{2(4n^2+8n+3) - 3(n^2+3n+2)}{4(n+1)(n+2)}$$

$$= \frac{8n^2+16n+6-3n^2-9n-6}{4(n+1)(n+2)}$$

$$= \frac{5n^2+7n}{4(n+1)(n+2)} = \frac{n(5n+7)}{4(n+1)(n+2)}$$

4

Use the midpoint formula wth $h = 0.1$ to estimate the value at $x = 0.2$ of the particular solution to

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{e^x + y}{y + x^2}\ at(0,1)$$

correct to 4 decimal places.

Euler's iterative formula

$$y_{n+1} \approx y_n + h\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_n$$

Midpoint iterative formula

$$y_{n+1} \approx y_{n-1} + 2h\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_n$$

$$y_1 \approx y_0 + h\left(\frac{dy}{dx}\right)_0 \qquad y_0 = 1 \qquad x_0 = 0$$

$$\left(\frac{dy}{dx}\right)_0 = \frac{e^0 + 1}{1 + 0} = 2$$

$$y_1 \approx 1 + 0.1\,(2)$$

$$y_1 = 1.2 \qquad x_1 = 0.1$$

$$\left(\frac{dy}{dx}\right)_1 = \frac{e^{0.1} + 1.2}{1.2 + (0.1)^2} = 1.9051$$

$$y_2 \approx y_0 + 2h\left(\frac{dy}{dx}\right)_1$$

$$y_2 \approx 1 + 2(0.1)(1.9051)$$

$$\underline{\underline{y_2 = 1.3810}}$$

5

(a) Show that $P(5,5,3)$ and $Q(-1,2,-3)$ are on opposite sides of the plane

$$\Pi_1 : 2x - 3y + 6z = 0$$

(b) Find where PQ meets the plane Π_1.

(c) Find the equation of the plane which contains the line PQ and is perpendicular to Π_1

a)

$$\underset{\sim}{r} \cdot \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} = 0$$

$$\begin{pmatrix} 5 \\ 5 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} = 10 - 15 + 18 = \underline{\underline{13}}$$

$$\begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} = -2 - 6 - 18 = \underline{\underline{-26}}$$

Hence opposite sides of plane.

b) PQ: $\underset{\sim}{r} = \begin{pmatrix} 5 \\ 5 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ 3 \\ 6 \end{pmatrix}$

$$\begin{pmatrix} 5 + 6\lambda \\ 5 + 3\lambda \\ 3 + 6\lambda \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} = 10 + 12\lambda - 15 - 9\lambda + 18 + 36\lambda$$

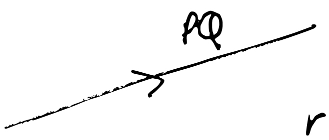
$$= 13 + 39\lambda$$

$$13 + 39\lambda = 0$$

$$\lambda = -\frac{1}{3}$$

Meets plane at $\begin{pmatrix} 5 \\ 5 \\ 3 \end{pmatrix} + -\frac{1}{3} \begin{pmatrix} 6 \\ 3 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$ $\quad \underline{\underline{(3,4,1)}}$

c)



$$\underset{\sim}{r} = \begin{pmatrix} 5 \\ 5 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ 3 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$$