The Complete Mathematics Conference

Karen Hancock

Worked Examples: The Power of Self-Explanation

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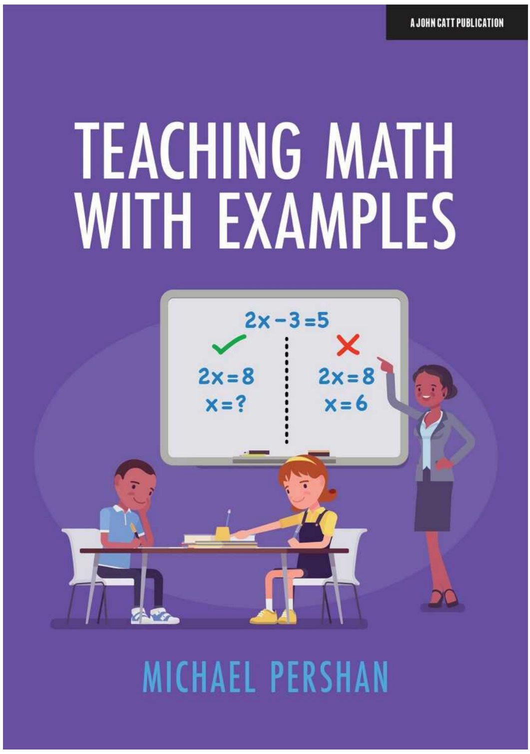
What inspired this?

https://www.ollielovell.com/errr/michaelpershan/



http://www.mrbartonmaths.com/blog/michael-pershan-teaching-with-worked-examples-part-2/

What is a worked example?

"Worked examples are completed solutions that we ask students to study and learn from."

Key points

- Students need to study the solution - not superficially
- Self explanation prompts should encourage students to generalise

The before

Factorise $6x + 9$

Factorise $25ab + 30b$

Factorise $30x^2 - 12x$

Your turn

Factorise $10x + 35$

Factorise $21cd + 14c$

Factorise $42y^2 - 49y$

Factorise $21f + 7$

Factorise $15r - 5$

In the classroom

- Display problem
- Reveal solution
- Quiet time to study
- Carefully explain to partner
- Self explanation prompts
- Your turn

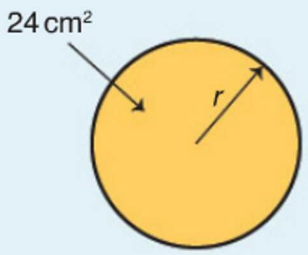
Different from text book

Off load the explanation from the solution

Text book examples

Example 2

The area of a circle is $24\,\text{cm}^2$. Find the radius.



Using $A = \pi r^2$

$24 = \pi r^2$ (Make r the subject of the equation)

$r^2 = \frac{24}{\pi}$

$r = \sqrt{\frac{24}{\pi}}$

$= 2.76\,\text{cm}$ to 3 s.f.

a $\sqrt{2}(5 - \sqrt{3})$ — $\sqrt{2} \times 5 - \sqrt{2} \times \sqrt{3}$

$= 5\sqrt{2} - \sqrt{2}\sqrt{3}$

$= 5\sqrt{2} - \sqrt{6}$ — Using $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

b $(2 - \sqrt{3})(5 + \sqrt{3})$

$= 2(5 + \sqrt{3}) - \sqrt{3}(5 + \sqrt{3})$ — Expand the brackets completely before you simplify.

$= 10 + 2\sqrt{3} - 5\sqrt{3} - \sqrt{9}$ — Collect like terms: $2\sqrt{3} - 5\sqrt{3} = -3\sqrt{3}$; Simplify any roots if possible: $\sqrt{9} = 3$

$= 7 - 3\sqrt{3}$

My journey - 1st use

WORKED EXAMPLE:

Simplify

$$\frac{6x}{10x^2}$$

$$= \frac{6 \times x}{10 \times x \times x}$$

$$= \frac{{}^{3}\cancel{6} \times \cancel{x}^{1}}{\cancel{10}_{5} \times \cancel{x}_{1} \times x}$$

$$= \frac{3 \times 1}{5 \times 1 \times x} = \frac{3}{5x}$$

Your turn

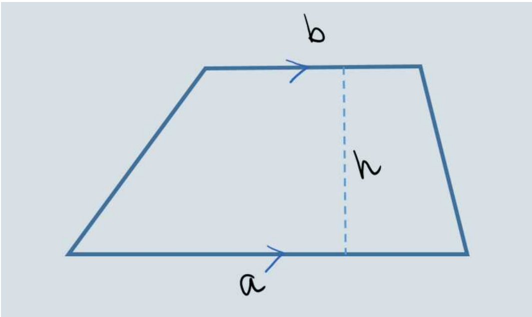
Simplify

$$\frac{8x^2}{20x^5}$$

1) Why have the 6 and the 10 been replaced with 3 and 5?
2) Why is it important to put the 1s in to replace the xs?

What if the question was $\frac{5x}{10x^2}$?

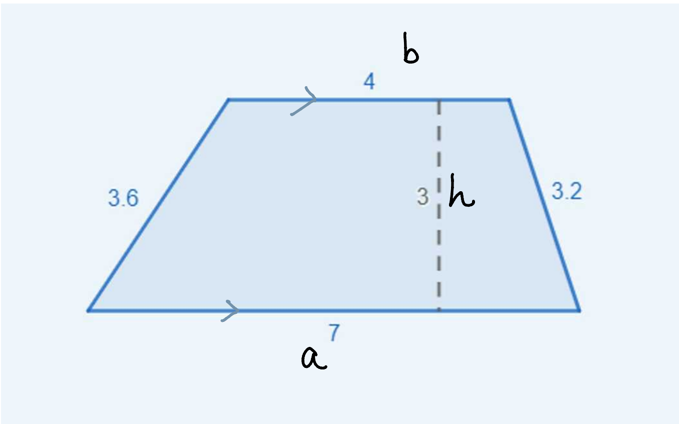
Attempt 2



Area = $\frac{1}{2}(a+b)h$

Worked Example:

Find the area of this trapezium:



$$\text{Area} = \frac{1}{2}(a+b)h$$

$$= \frac{1}{2}(7+4)(3)$$

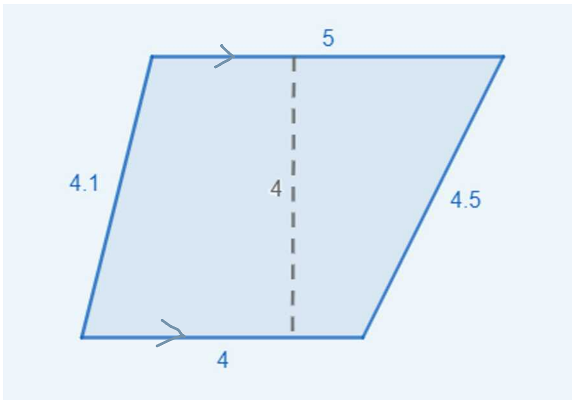
$$= \frac{1}{2}(11)(3)$$

$$= \underline{\underline{16.5}}$$

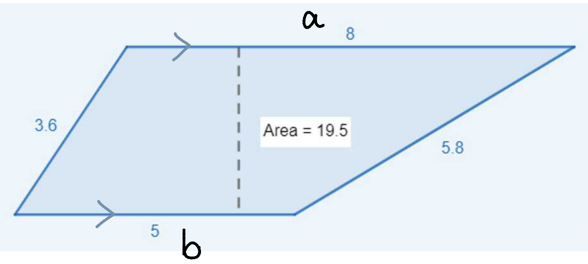
- How did we get an answer of 16.5 from $\frac{1}{2}(11)(3)$?
- Why do we ignore the 3.6 and 3.2 on the diagram?
- When might it be useful to know the 3.6 and the 3.2?

Your turn:

Find the area of this trapezium:



Calculate the height of this trapezium:



$$Area = \frac{1}{2}(a+b)h$$

$$19.5 = \frac{1}{2}(8+5)h$$

$$19.5 = \frac{1}{2}(13)h$$

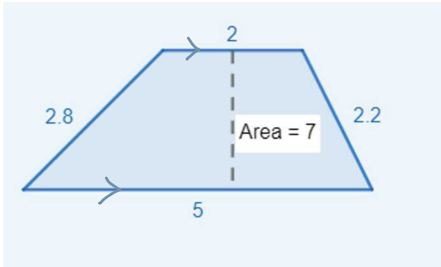
$\times 2$ $\times 2$

$$39 = 13h$$

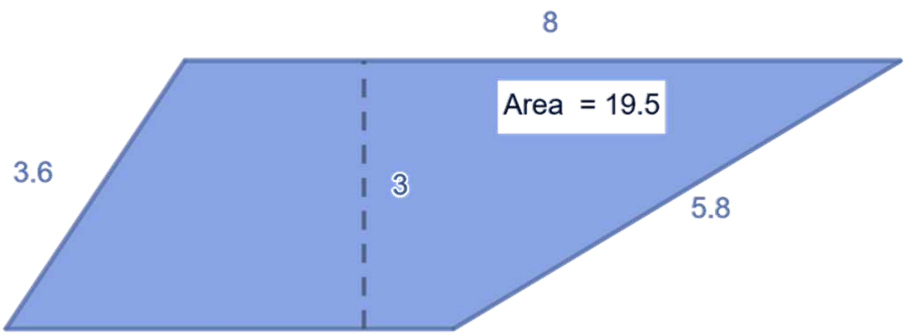
$\div 13$ $\div 13$

$$3 = h$$

Calculate the height of this trapezium:



- Could you think of another way of finishing this question from $19.5 = \frac{1}{2}(13)h$?



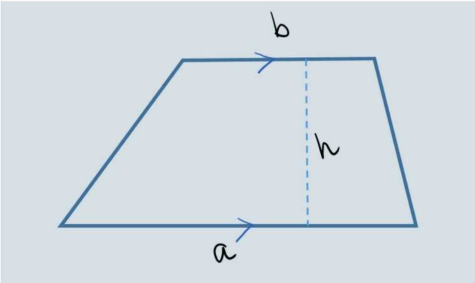
How would your solution to this question look different?

Area of Trapezium example

Area of Trap
example

Area of a trapezium 28/04/2021

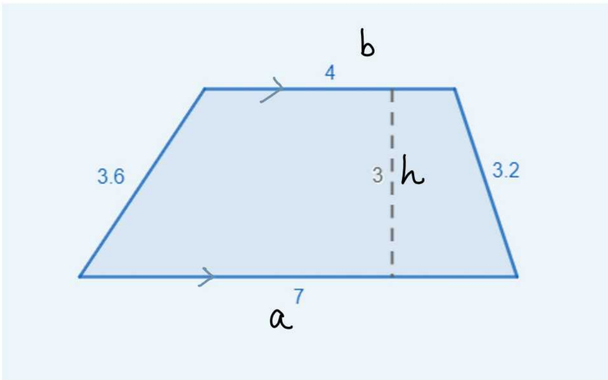
https://www.youtube.com/watch?v=qlxawNewXiY



Area = $\frac{1}{2}(a + b)h$

Worked Example:

Find the area of this trapezium:

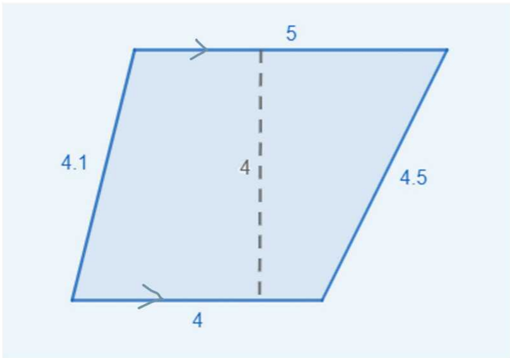


$$\text{Area} = \frac{1}{2}(a+b)h$$
$$= \frac{1}{2}(7+4)(3)$$
$$= \frac{1}{2}(11)(3)$$
$$= \underline{\underline{16.5}}$$

- How did we get an answer of 16.5 from $\frac{1}{2}(11)(3)$?
- Why do we ignore the 3.6 and 3.2 on the diagram?
- When might it be useful to know the 3.6 and the 3.2?

Your turn:

Find the area of this trapezium:



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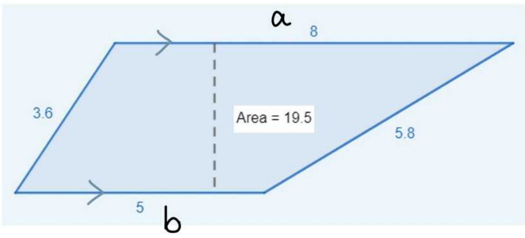


$$\text{Area} = \frac{1}{2}(a+b)h$$

$$19.5 = \frac{1}{2}(8+5)h$$

$$19.5 = \frac{1}{2}(13)h$$

$\times 2$ $\times 2$

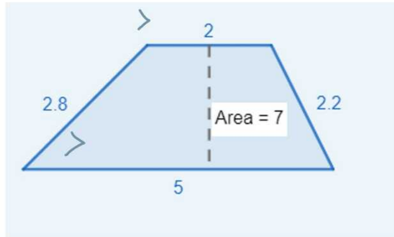
$$39 = 13h$$

$\div 13$ $\div 13$

$$3 = h$$

- Could you think of another way of finishing this question from $19.5 = \frac{1}{2}(13)h$?

Calculate the height of this trapezium:



Self explanation prompts - improving the questions.

In my own teaching I lately find myself relying a lot on "what if" questions to generate generalizations

From <https://twitter.com/mpershan/status/1384242050455916551>

Attempt 3

Worked Example:

Increase £450 by 30%

10% of £450 = £45

30% of £450 = 45×3

= £135

∴ 450+135 = £585

Your Turn:

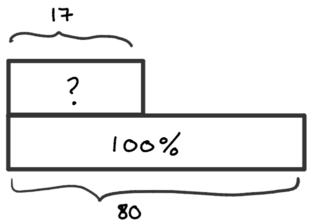
Decrease £140 by 23%

- What if the question had said **decrease**?
- What if the question had said **increase by 100%**?

Attempt 4

Worked example

Write 17 as a percentage of 80



$$\frac{100}{80} \times 17 = \frac{10 \times \cancel{10} \times 17}{8 \times \cancel{10}}$$

$$= \frac{170}{8} = \frac{85}{4} = 21.25\%$$

$$4 \overline{)85.00} = 21.25$$

- How could you check that your answer was about the right size?
- If the question was "Write 17 as a percentage of 50", would you approach it in the same way?

Your turn

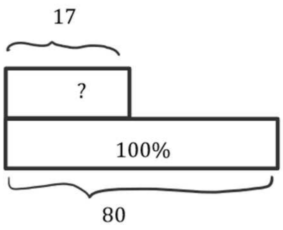
Write 11 as a percentage of 40

Student handout

One number as a percentage of another

Worked example

Write 17 as a percentage of 80



$$\frac{100}{80} \times 17 = \frac{10 \times 10 \times 17}{8 \times 10}$$

$$= \frac{170}{8} = \frac{85}{4} = 21.25\%$$

$$4\,\overline{)\,85.00}\quad = 21.25$$

- How could you check that your answer was about the right size?

- If the question was "Write 17 as a percentage of 50", would you approach it in the same way?

Your turn

Write 11 as a percentage of 40

Attempt 5

19 May 2021 16:43

Worked example:

Find the highest common factor of 196 and 112

196

$= 2 \times 98$

$= 2 \times 2 \times 49$

$= 2 \times 2 \times 7 \times 7$

112

$= 2 \times 56$

$= 2 \times 7 \times 8$

$= 2 \times 7 \times 2 \times 4$

$= 2 \times 7 \times 2 \times 2 \times 2$

To find HCF

$196 = \underline{2} \times \underline{2} \qquad \times \underline{7} \times 7$

$112 = \underline{2} \times \underline{2} \times 2 \times 2 \times \underline{7}$

$HCF = 2 \times 2 \times 7$

$= 4 \times 7$

$= \underline{\underline{28}}$

- Why is the multiplication written with gaps?

- What if there were no primes in common? (Eg 25 and 18)

Your turn

1. Find the highest common factor of 252 and 210

2. Find the highest common factor of 288 and 441

Attempt 6

19 May 2021 16:43

Worked example:

Find the lowest common multiple of 1650 and 234

Find the LCM of

1650 and 234

1650

$= 5 \times 330$

$= 5 \times 3 \times 110$

$= 5 \times 3 \times 2 \times 55$

$= 5 \times 3 \times 2 \times 5 \times 11$

234

$= 2 \times 117$

$= 2 \times 3 \times 39$

$= 2 \times 3 \times 3 \times 13$

To find LCM

$1650 = 11 \times 5^2 \times 3 \times 2$

$234 = 13 \times 3^2 \times 2$

$LCM = 13 \times 11 \times 5^2 \times 3^2 \times 2$

- Why haven't they ringed the 2 in both compositions?
- What if one of the decompositions contained 2^3?

Your turn

1. Find the lowest common multiple of 252 and 1470

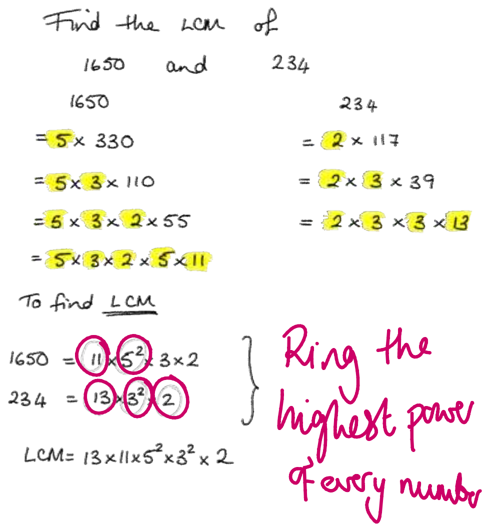
2. Find the lowest common multiple of 288 and 147

With added annotation

19 May 2021 16:43

Worked example:

Find the lowest common multiple of 1650 and 234



- Why haven't they ringed the 2 in both compositions?
- What if one of the decompositions contained 2^3?

Your turn

1. Find the lowest common multiple of 252 and 1470

2. Find the lowest common multiple of 288 and 147

Attempt 7

Worked Example:

A youth club has 60 members.

40 are boys.

20 are girls.

The members are surveyed on the number of films they watched last week.

The mean number watched by the boys was 3.3

The mean number watched by the girls was 1.8

What is the mean number watched by the club members?

3.3 films 40 boys $\Rightarrow$ $40 \times 3.3 = 132$ films

1.8 films 20 girls $\Rightarrow$ $20 \times 1.8 = 36$ films

Total = 168 films

Mean = $\frac{168}{60} = 2.8$ films

- Why is it wrong to find the total mean by calculating $\frac{3.3+1.8}{2}$?
- How would the answer change it the boys' average was 1.8, and the girls' average was a 3.3?

Your Turn:

A group contains 12 girls and 8 boys.

The average age of the boys is 12.125 years

The average age of the girls is 8.25 years

What is the average age of the group

Attempt 8

Worked Example:

Calculate the circumference of a circle with diameter 12cm

$C = \pi d \quad d = 12$

$C = \pi(12)$

$C = 12\pi$

$C = \underline{\underline{37.7cm}}$ (3s.f)

What if you had been told the radius was 8cm?
How might you approach the question?

Why might it be useful to leave your answer as 12π cm?

Your Turn:

Calculate
showing ALL your working
the circumference of a circle with RADIUS 4.3cm

Give your answer to 2 decimal places

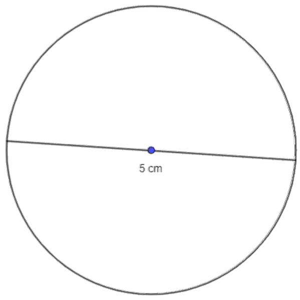
Attempt 9

18 May 2021 19:53

Area of a circle = πr^2

Worked example:

Calculate the area of the following circle:



$A = \pi r^2$

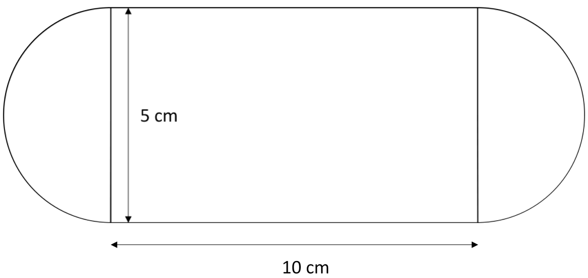
$d = 5 \quad r = 2.5$

$A = \pi (2.5)^2$

$= \frac{25}{4}\pi$

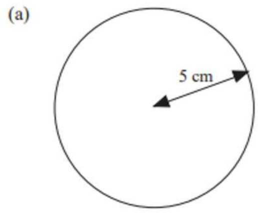
$= 19.63 cm^2$

- Why in the example do they only square the 2.5?
- What happens to the area if the diameter is doubled?
- What if you were asked to find the area of this shape next - can you see any shortcuts?

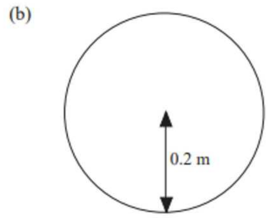


Your turn

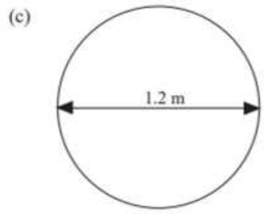
Calculate the area of the following circles



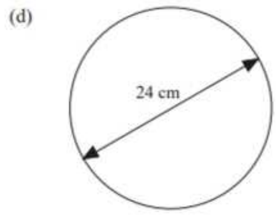
$25\pi = 78.5cm^2$



$\frac{1}{25}\pi = 0.13m^2$



$\frac{9}{25}\pi = 1.13m^2$



$144\pi = 452.4cm^2$

Working live - but using prompts

15 May 2021 16:50

Worked Example:

1) Calculate an estimate for the mean of this data:

Height	Frequency		
$120 < h \leq 130$	3		
$130 < h \leq 140$	5		
$140 < h \leq 150$	2		
$150 < h \leq 160$	6		
$160 < h \leq 170$	1		
$170 < h \leq 180$	1		

2) Write down the modal class

3) Which group does the median lie in?

- Why are we calculating an **estimate** for the mean?

- If you calculated the estimate of the mean as 435, how could you easily spot you were wrong?

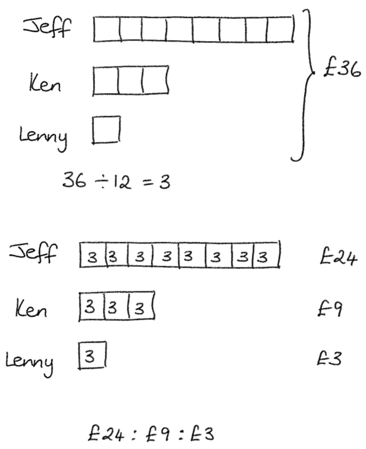
- If we included a new value in the group $160 < h \leq 170$ would the mean:
  - Increase
  - Decrease
  - Stay the same

Ratio - twice over

Year 8

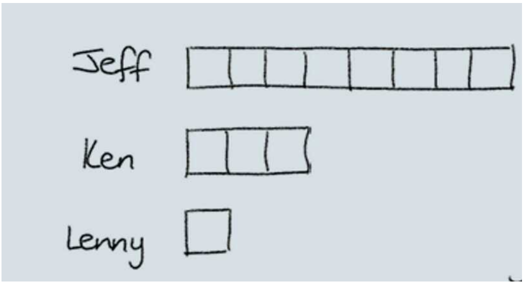
Worked example:

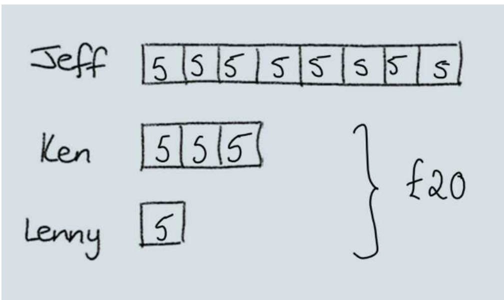
Share £36 between Jeff, Ken and Lenny in the ratio 8:3:1



- How can I quickly check whether I have made a mistake?

- What if I was told that altogether Ken and Lenny received £20, how would that make the question (and answer different)?





Your turn:

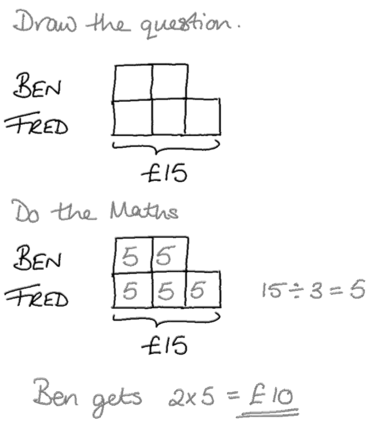
Divide £240 between Anna, Bess and Charlie in the ratio 5:2:1

Year 9

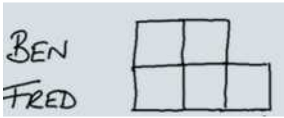
Worked Example:

Ben and Fred share some money in the ratio 2:3.

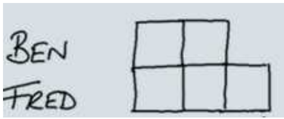
Fred gets £15, how much does Ben get?



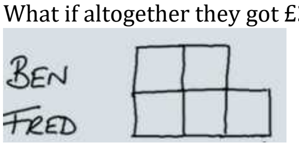
- What if Ben got the £15, how much would Fred get?



- What if Fred got £8 more than Ben?



- What if altogether they got £30?



Your Turn

In a recipe the ratio of butter to sugar to flour is 2:1:4.

I use 25g of butter, how much flour and sugar do I need?

Revision - faded examples

08 June 2021 17:22

Standard Deviation recap

Standard deviation = $\sqrt{\frac{1}{n}\sum(x-\bar{x})^2}$

An alternative formula for standard deviation is

standard deviation = $\sqrt{\frac{\sum x^2}{n}-\left(\frac{\sum x}{n}\right)^2}$

Complete this question:

Calculate the standard deviation of the following data set:

3, 4, 7, 5, 6, 8, 3, 4, 3, 2

$\bar{x} = \frac{\sum x}{n} = \frac{3+4+7+5+6+8+3+4+3+2}{10}$

$= \frac{45}{10} = 4.5$

$\sum x^2 = 3^2+4^2+7^2+5^2+6^2+8^2+3^2+4^2+3^2+2^2$

$= 9+16+49+25+36+64+9+16+9+4$

= [redacted]

$\sigma_x = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$

= $\sqrt{\frac{\text{[redacted]}}{10} - (4.5)^2}$

= $\sqrt{\text{[redacted]}}$

= [redacted]

Now do this question:

Calculate the standard deviation of the following data set:

12, 14, 12, 10, 7, 8, 3, 14

Read this example

Calculate the standard deviation of the following data set:

You can assume $\sum fx^2 = 329$

x	f	fx
1	5	5
2	7	14
3	8	24
4	14	56
	34	99

$\bar{x} = \frac{99}{34} = 2.911$

$$\sigma_x = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

$$= \sqrt{\frac{329}{34} - \left(\frac{99}{34}\right)^2}$$

$$= \sqrt{1.198}$$

$$= \underline{\underline{1.09}}$$

Check you can get this number on your calculator.
1.09457612...

Now do this question:

Calculate the standard deviation of the following data set

You may assume $\sum fx^2 = 1177$

x	f
4	2
5	4
6	8
7	1
8	6
9	4

Complete this question:

Calculate the standard deviation of the following data set:

3, 4, 7, 5, 6, 8, 3, 4, 3, 2

$$\bar{x} = \frac{\Sigma x}{n} = \frac{3+4+7+5+6+8+3+4+3+2}{10}$$

$$= \frac{45}{10} = \underline{\underline{4.5}}$$

$$\Sigma x^2 = 3^2 + 4^2 + 7^2 + 5^2 + 6^2 + 8^2 + 3^2 + 4^2 + 3^2 + 2^2$$

$$= 9 + 16 + 49 + 25 + 36 + 64 + 9 + 16 + 9 + 4$$

= [redacted]

$$\sigma_x = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$$

= √([redacted] / 10 − $(4.5)^2$)

= √ [redacted]

= [redacted]

Now do this question:

Calculate the standard deviation of the following data set:

12, 14, 12, 10, 7, 8, 3, 14

Read this example:

Calculate the standard deviation of the following data set:

You can assume that $\sum fx^2 = 329$

x	f	fx
1	5	5
2	7	14
3	8	24
4	14	56
	34	99

$\bar{x} = \frac{99}{34} = 2.911$

$\sigma_x = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$

$= \sqrt{\frac{329}{34} - \left(\frac{99}{34}\right)^2}$

$= \sqrt{1.198}$

$= 1.09$

Check you can get this number on your calculator.
1.09457612...

Now do this question:

Calculate the standard deviation of the following data set

You may assume $\sum fx^2 = 1177$

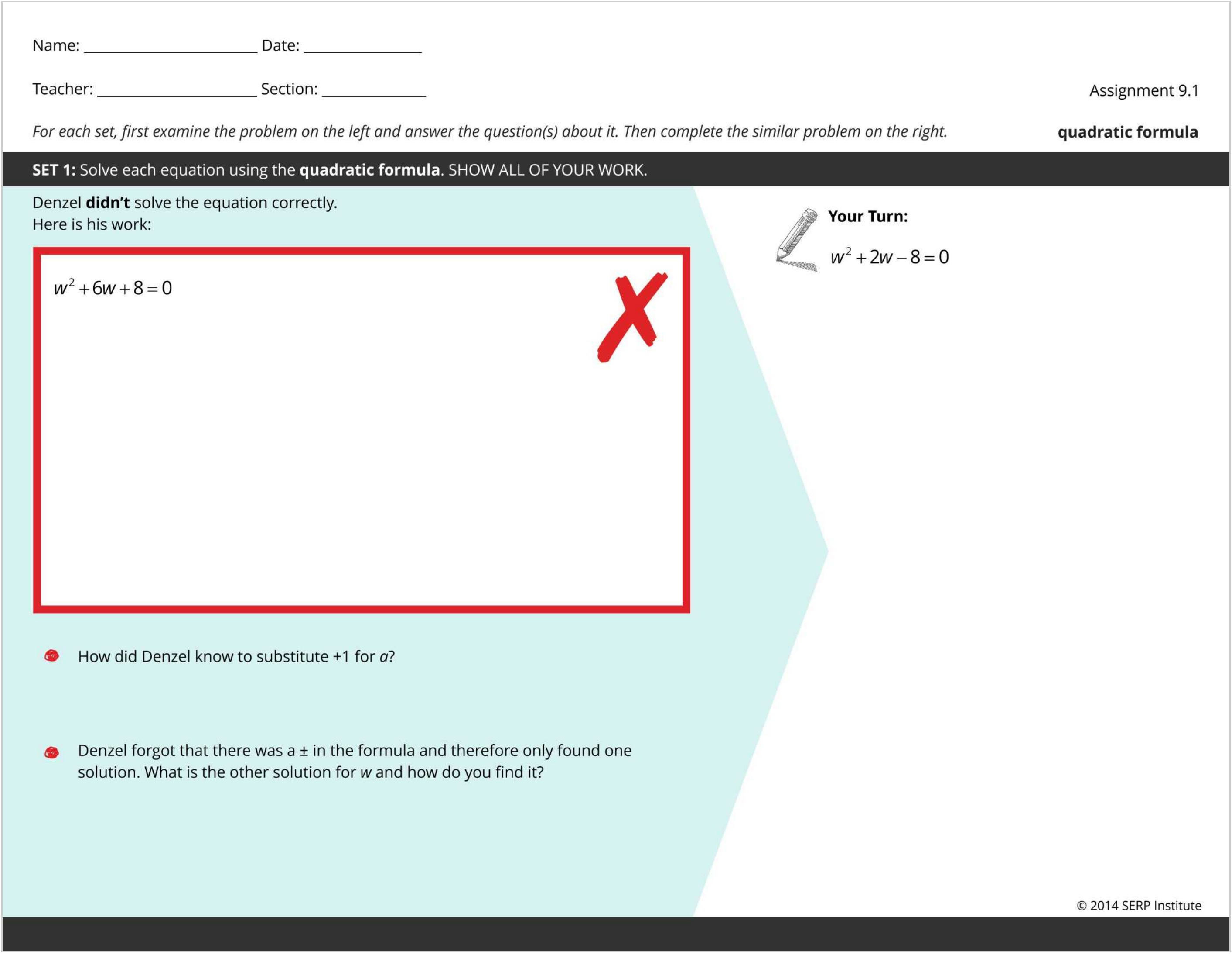
x	f
4	2
5	4
6	8
7	1
8	6
9	4

Places to find prompts/examples

SERP Institute: https://www.serpinstitute.org/educator-resources

Algebra By Example
Math By Example

9.1_abe_w



Name: ____________________ Date: _____________

Teacher: ___________________ Section: ____________

Assignment 9.1

For each set, first examine the problem on the left and answer the question(s) about it. Then complete the similar problem on the right.

quadratic formula

SET 1: Solve each equation using the **quadratic formula**. SHOW ALL OF YOUR WORK.

Denzel **didn't** solve the equation correctly.
Here is his work:

$w^2 + 6w + 8 = 0$

- How did Denzel know to substitute +1 for *a*?

- Denzel forgot that there was a ± in the formula and therefore only found one solution. What is the other solution for *w* and how do you find it?

Your Turn:

$w^2 + 2w - 8 = 0$

SET 2: Solve each equation using the **quadratic formula**. SHOW ALL OF YOUR WORK.

Abdalla solved this equation **correctly**.
Here is his work.

$4w^2 - 4w = -1$

- Why did Abdalla +1 to both sides before applying the quadratic formula?

- Why is there only one solution to this equation?

Your Turn:

$9w^2 + 12w = -4$

SET 3: Solve each equation using the **quadratic formula**. SHOW ALL OF YOUR WORK.

Maya solved this equation **correctly**.
Here is her work:

$-5 = x^2 + 5x$

- Would Maya have gotten the same answer if she had moved x^2 and $5x$ to the left hand side in the first step instead of moving –5 to the right hand side? Explain why or why not.

- Why are –1.38 and –3.62 approximate solutions (≈) ?

Your Turn:

$1 + 3x^2 = -5x$

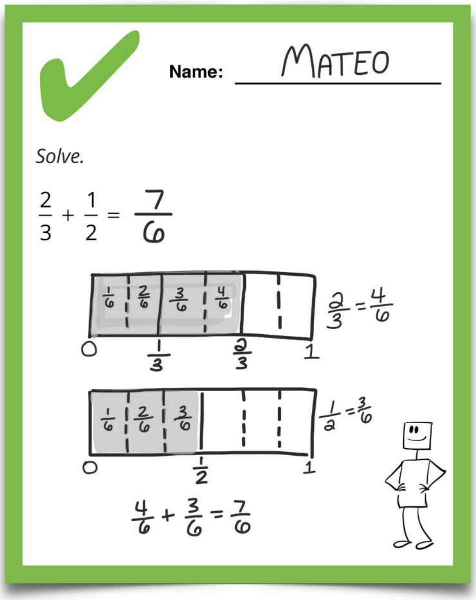
3.-Adding-Fractions...

Name: ______________________ Date: _______________

MathByExample
Basic Operations with Fractions
Adding Fractions with Unlike Denominators
CCSS 5.NF.A.1

SET ONE

1. Study Mateo's correct work.



2. Answer the question.

- Why did Mateo find equivalent fractions for $\frac{2}{3}$ and $\frac{1}{2}$?

3. Then complete this one.

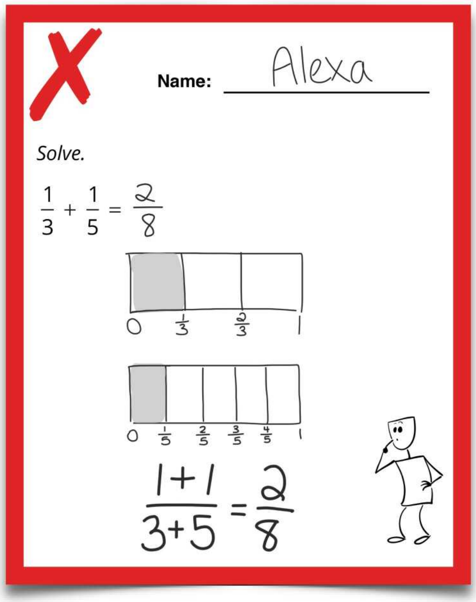
Solve.

$\frac{2}{5} + \frac{1}{2} =$

Adding Fractions with Unlike Denominators, continued

SET TWO

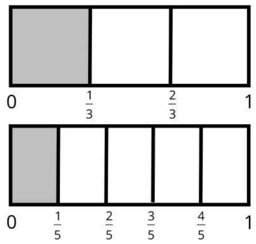
1. Study Alexa's incorrect work.



2. Answer these questions.

- What mistake did Alexa make when adding the fractions?

- Help Alexa by partitioning the diagrams below into an equal number of parts in order to find a common denominator.



- Based on the diagrams above, what fraction is $\frac{1}{3}$ equivalent to?

3. Then complete this one.

Solve.

$\frac{1}{3} + \frac{1}{4} =$

Prompt ideas

https://www.researchgate.net/publication/281066179_A_Worked_Example_for_Creating_Worked_Examples

Mcginn, Kelly & Lange, Karin & Booth, Julie. (2015). A Worked Example for Creating Worked Examples. Mathematics Teaching in the Middle School. 21. 27 - 33. 10.5951/mathteacmiddscho.21.1.0026.

Sample Prompts

Although it is acceptable to ask procedural questions, be sure to ask students to explain and/or justify their reasoning.

1. Why is __________ not included in the answer?
2. What did [student name] __________ as his first step?
3. What should [student name] have done to __________?
4. Would it have been OK to write __________? Why or why not?
5. Why did [student name] combine __________ and __________?
6. Why did [student name] first _______ then _______?
7. Is _______ the same expression as _______? Explain.
8. Would [student name] have gotten the same answer if he [or she] _______ first?
9. Why did [student name] change __________ to __________?
10. Explain why __________ would have been an unreasonable answer.
11. How could [student name] have figured out that his [or her] answer did not make sense?
12. How did [student name] know that __________ was not equal to __________?
13. What did the __________ represent in this word problem?
14. How did the __________ in the equation affect the graph?
15. Why did [student name] __________ from both sides of the equation?

Correcting Homework

Identify the questions most of them struggled on:

8.	**Angle properties**
	Find the size of the largest exterior angle in the polygon shown.

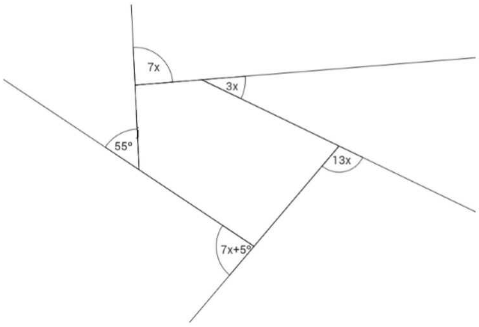
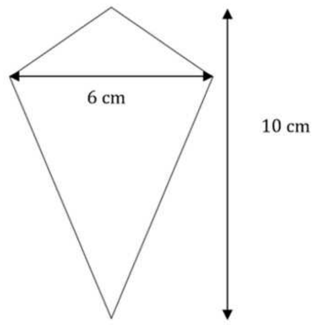


Diagram not drawn to scale

....(3)

9.	**Area**
	Find the area of the kite shown  **Diagram not drawn to scale** ....(2)

4.	**Straight Line Graphs**
	Calculate the midpoint of the line joining the points (9,1) and (-3, 5) ....(2)
5.	**Straight Line Graphs**
	Calculate the gradient of the line joining the points (9,1) and (-3, 5) ....(2)

HOMEWORK SHEET 17-1

WORKED EXAMPLES

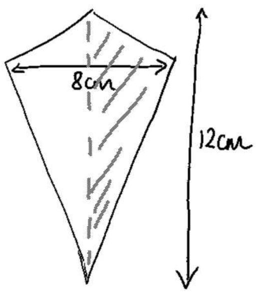
Find the midpoint of (4,5) and (7, -3)

$x_1 \; y_1 \quad x_2 \; y_2$

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{4+7}{2}, \frac{5+-3}{2}\right)$$

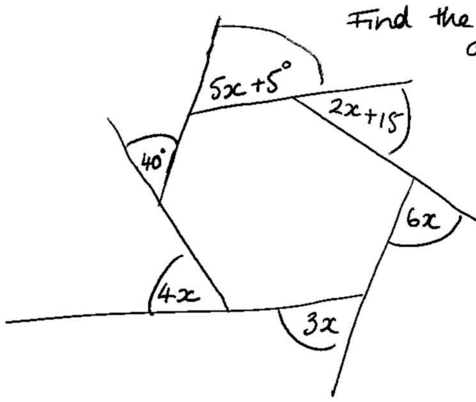
$$= (5.5, 1)$$



$\frac{1}{2}(4)(12) = 24$

$2 \times 24 = 48\text{cm}^2$

Find the largest angle.



$$4x + 40 + 5x + 5 + 2x + 15 + 6x + 3x = 360$$

$$20x + 60 = 360$$

$$20x = 300$$

$$x = 15$$

LARGEST ANGLE $= 6x = 6 \times 15 = 90°$

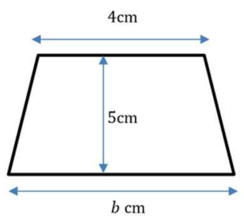
Gradient of line joining (4,5) and (7,-3)

$$\frac{y_2-y_1}{x_2-x_1} = \frac{-3-5}{7-4} = -\frac{8}{3}$$

Hints for homework

17 May 2021 21:33

7.	**Unit 14**
	Write these numbers in numerical order: $0.00358, 3.5 \times 10^{-4}, 0.0378, 3.8 \times 10^{-2}, 3.8 \times 10^{0}, 3.2$
8.	**Unit 15**
	The bearing of B from A is075°. The bearing of C from B is 175°. The distance AB=AC. Draw a sketch and find the bearing of A from C.
9.	**Unit 16**
	The area of the trapezium shown is 27.5cm^2. Find the length of side b.



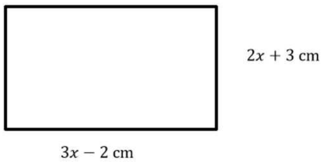
Worked Example:

The bearing of B from A is 050°

The bearing of C from B is 160°

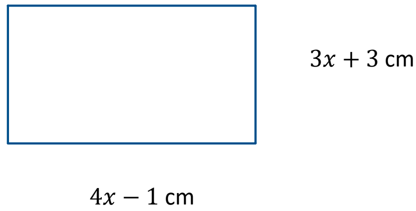
The distance $AB = AC$

Draw a sketch and find the bearing of A from C

10.	**Unit 17**
	The perimeter of the rectangle shown is less than 14 cm. Find the range of values for x.  HINT: Both sides also need to bigger than zero.
11.	**Unit 18**
	The mean of a set of 5 numbers is 4.4 The largest number is 10. The range is 15. The median is 8. The mode is 10. Write down the 5 numbers.

Worked Example:

The perimeter of the rectangle shown is less than 20 cm. Find the range of values for x



Worked Example:

The mean of a set of 5 numbers is 8.2
The largest number is 21
The range is 22
The median is 9
The mode is 9

Write down the 5 numbers

Hints for homework

17 May 2021 21:03

7. **Unit 14**

Write these numbers in numerical order:

$0.00350, 3.5 \times 10^{-4}, 0.0378, 3.0 \times 10^{-2}, 3.8 \times 10^{0}, 3.2$

8. **Unit 15**

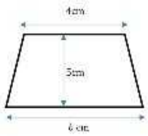
The bearing of B from A is 075°. The bearing of C from B is 175°. The distance AB=AC.

Draw a sketch and find the bearing of A from C.

9. **Unit 16**

The area of the trapezium shown is 27.5cm².

Find the length of side b.



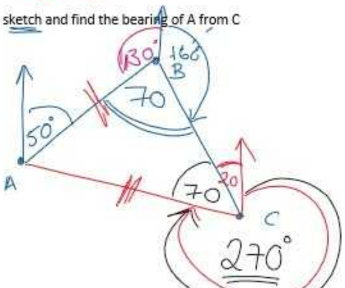
Worked Example:

The bearing of B from A is 050°

The bearing of C from B is 160°

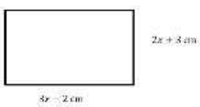
The distance $AB = AC$

Draw a sketch and find the bearing of A from C



10. **Unit 17**

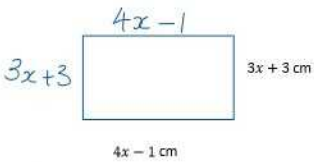
The perimeter of the rectangle shown is less than 14 cm. Find the range of values for x.



HINT: Both sides also need to bigger than zero.

Worked Example:

The perimeter of the rectangle shown is less than 20 cm. Find the range of values for x



All sides must bigger zero

Perimeter

$P = 4x - 1 + 3x + 3 + 4x - 1 + 3x + 3$

$= 14x + 4$

$14x + 4 < 20$

$14x < 16$

$x < 16/14$

Sides

$4x - 1 > 0$

$4x > 1$

$x > 1/4$

$1/4 < x < 16/14$

11. **Unit 18**

The mean of a set of 5 numbers is 4.4.

The largest number is 10.

The range is 15.

The median is 8.

The mode is 10.

Write down the 5 numbers.

Worked Example:

The mean of a set of 5 numbers is 8.2

The largest number is 21

The range is 22

The median is 9

The mode is 9

Write down the 5 numbers

$Total = 8.2 \times 5$

$= 41$

| -1 | 3 | 9 | 9 | 21 |

$-1 + 9 + 9 + 21 = 38$

Next Steps

Planning for September - A Level notes

Examples they have seen before:

Example

Rationalise the denominator on

$$\frac{3+\sqrt{5}}{4-\sqrt{5}}$$

$$= \frac{3+\sqrt{5}}{4-\sqrt{5}} \times \frac{4+\sqrt{5}}{4+\sqrt{5}}$$

$$= \frac{(3+\sqrt{5})(4+\sqrt{5})}{(4-\sqrt{5})(4+\sqrt{5})}$$

$$= \frac{12+4\sqrt{5}+3\sqrt{5}+5}{16-5}$$

$$= \frac{17+7\sqrt{5}}{11}$$

Why do we use $\frac{4+\sqrt{5}}{4+\sqrt{5}}$ as a multiplier?

What if the denominator was $\sqrt{3}+\sqrt{7}$, how would that change your method?

Practice – Non Calculator

1) Rationalise the denominator on

$$\frac{\sqrt{5}}{6-3\sqrt{5}}$$

2) Rationalise the denominator on

$$\frac{4+\sqrt{7}}{3+\sqrt{7}}$$

3) Rationalise the denominator on

$$\frac{\sqrt{3}-\sqrt{11}}{2\sqrt{3}+4\sqrt{11}}$$

TASK: Ex 1F Page 15

Planning for September - A Level Notes

New content

Disguised quadratics:

Example:

Solve

$$x^4 - 7x^2 = 12 = 0$$

Let $y = x^2$

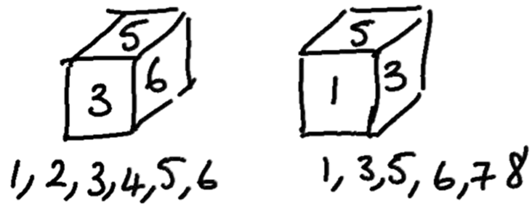
$\Rightarrow \quad y^2 - 7y + 12 = 0$

Complete question from here – leave your answers as surds

What substitution would you use if the question was $x - 7\sqrt{x} + 12 = 0$?

What would be your first step in solving $\frac{1}{x^2} - \frac{7}{x} + 12 = 0$

Worked Example:



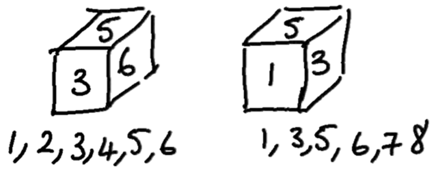
The two dice shown are rolled and the **total** of their rolls is worked out.

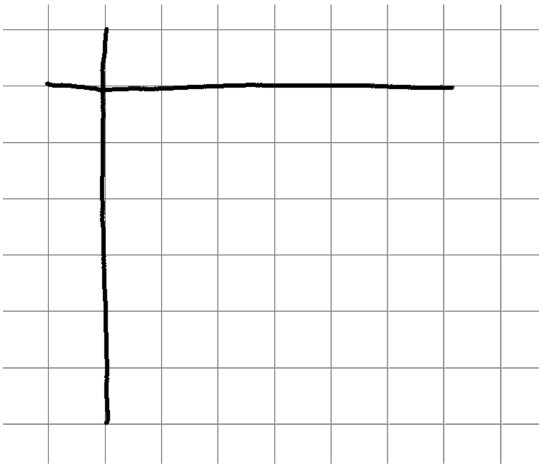
Use a sample space to find the probability that the total of the two dice is 9.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
3	4	5	6	7	8	9
5	6	7	8	9	10	11
6	7	8	9	10	11	12
7	8	9	10	11	12	13
8	9	10	11	12	13	14

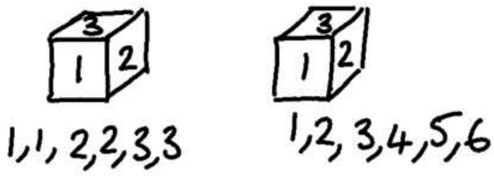
$P(9) = \frac{\quad}{36}$

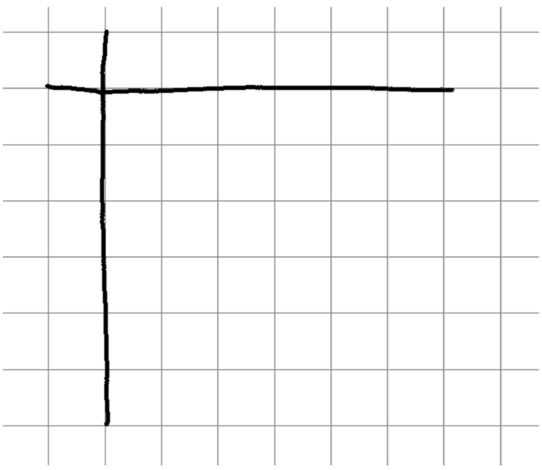
What would the sample space look like if the question involved the **product** of the two dice?





What would the sample space look like if the question involved the **total** of these two dice?





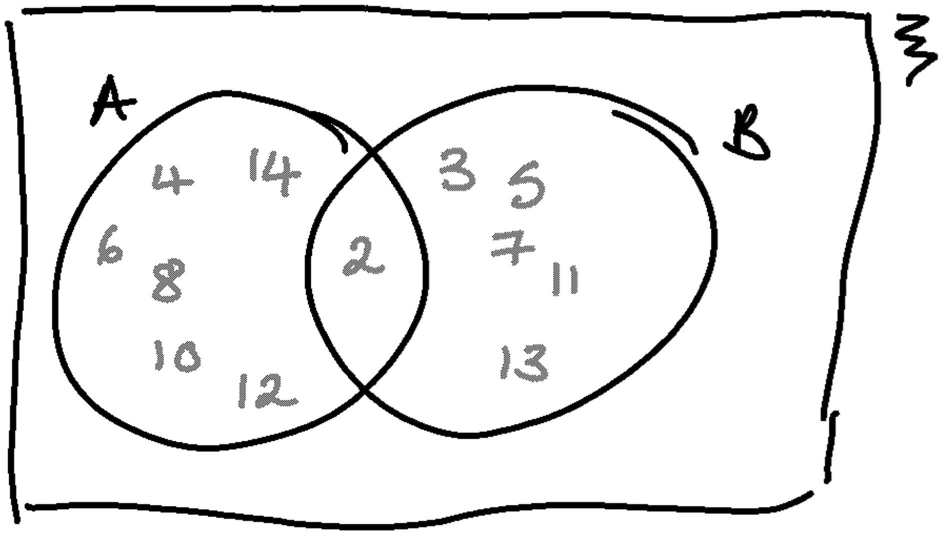
Planning for September - Incorrect/Incomplete examples

Worked Example:

ξ = Integers less than 15
A = Even numbers
B = Prime numbers

Show this on a Venn diagram

Here is Huda's solution, **it is incomplete**



Why is Huda's solution incomplete?

What I have noticed

- Students talk about the Maths more to each other.

- I've been focusing much more on my presentation of solutions.

- Writing the self-explanation prompts is hard.

Top tips for getting started

- Focus on the students explaining the Maths to themselves.

- One prompt is fine, don't worry if you can't think of several

- Focus prompts on:

  - What happens if we change something?
  - Boundary examples/non-examples
  - Error checking
  - Common misconceptions

Any Questions?

- How do you encourage students to be thinking in the same way as your original prompts so the 'new' ones aren't quite so big of a step?

- When do you choose to use this vs other instruction as you mentioned before? What makes a topic good to teach by worked examples?

- What is the value in doing these prompts at this stage, vs asking these kind of questions after other introductions and fluency built up?

- How are you distinguishing the questions you are asking that are 'about the maths' in the way you want and the way you are saying you moved away from?

- Can you please confirm what you mean by a 'boundary' questions

- I sometimes get the pupils to ask a ‘What if’ question so they are providing the prompts themselves. It can work surprisingly well!

- Do you think there was value the 'lower level' ones to start, so they knew what to be discussing together & you could then move on to deeper ones?

- Before showing worked examples, how do you explain the topic or you don't at all?

The Complete Mathematics Conference

Karen Hancock

Worked Examples:
The Power of
Self-Explanation

@karenshancock